Mathematics

The Mathematics subject test would consist of multiple choice questions (~40) from the fundamental topics of mathematics. The main subjects include Calculus (I & II), Real and Complex Analysis, Set topology, Linear and abstract Algebra and Ordinary Differential Equations. Following is the detailed topic-wise list

Linear Algebra:

- Linear equations and matrices, reduction to row echelon form Vector spaces:
    1. Vector spaces, subspaces, quotient spaces
    2. Linearly independent sets
    3. Linear transformations
    4. Kernel and image
    5. Projections (idempotent linear operators)
    6. Bases and dimension for finite dimensional vector spaces.
- Matrices and linear transformations between finite dimensional vector spaces:
    1. The matrix of a linear transformation with respect to a choice of bases
    2. Similarity of matrices and change of basis for linear transformations
    3. The inverse of a matrix
    4. The determinant of a square matrix
    5. The characteristic polynomial 6. The minimal polynomial
    7. Eigenvalues and Eigenvectors
    8. Diagonalization
    9. Cayley-Hamilton theorem
    10. Rank + nullity = dimension of domain.
- Finite dimensional inner product spaces:
    1. The standard positive definite inner product on real n-space
    2. Length and angle
    3. Gram-Schmidt orthogonalization.

Abstract Algebra:

 Groups and Rings

1. Elementary concepts (homomorphism, subgroup, coset, normal subgroup)
2. Lagrange's Theorem
3. Cauchy's Theorem
4. Commutator Subgroup
5. Sylow theorems
6. Structure of finitely generated Abelian groups
7. Symmetric, alternating, dihedral

8. General linear groups, Commutative rings and ideals (principal, prime, maximal)
9. Integral domains, Euclidean domains, principal ideal domains
10. Subgroups of the Integers
11. Greatest Common Divisors
12. The Euclidean Algorithm
13. Prime Numbers
14. The Fundamental Theorem of Arithmetic
15. The Infinitude of Primes
16. Congruences

Analysis (including Calculus, Real Analysis, Complex Analysis and Set Topology):

- Metric Spaces
  1. Metric Spaces
  2. Convergence of sequences in metric spaces
  3. Cauchy sequences
  4. Completeness
  5. Contraction principle

- Topological spaces
  1. continuous maps
  2. Hausdorff spaces
  3. Compactness
  4. Connectedness

- The real numbers
  1. The real numbers
  2. The real numbers as a complete ordered field
  3. Closed bounded subsets are compact
  4. Intermediate value theorem
  5. Maxima and minima for continuous functions on a compact set

- Differentiation
  1. Differentiation of a function in one real variable
  2. Mean Value Theorem
  3. L'Hopital's Rule
  4. Taylor's Theorem with error estimates

- Riemann integration of functions in one real variable
  1. Riemann integrable functions
  2. Integration and anti-differentiation

- Sequences and series of functions
    1. Power series and interval of convergence
    2. Uniform convergence of sequences of functions
    3. Uniform convergence and integration

- Differential Calculus for functions from n-space to reals and reals to n-space
    1. Parametrized curves
    2. Tangent vectors
    3. Velocity
    4. Acceleration
    5. Partial derivatives
    6. Directional derivatives
    7. The gradient
    8. The chain rule
    9. Taylor's theorem
    10. Local maxima and minima
    11. Level surfaces of functions
    12. Tangent planes to surfaces in 3-space
    13. Lagrange multipliers

- Differential Calculus for functions from n-space to m-space
    1. Notion of derivative
    2. Chain rule
    3. Inverse function theorem
    4. Implicit function theorem

- Integral Calculus in several variables
    1. The integral, path and surface integrals
    2. Green's theorem in the plane
    3. The divergence theorem in 3-space
    4. The change of variables formula

- Complex Variables

    1. Complex Algebra and Complex Plane
    2. Analytic functions
    3. Cauchy’s theorem
    4. Cauchy integral formula

Ordinary Differential Equations:

1. First order differential equations

2. Second order linear differential equations
3. Systems of first order differential equations