Lahore University of Management Sciences

Differential Geometry/ Advanced Differential Geometry

Semester & Year

Instructor	Waqas Ali Azhar
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Course URL (if any)	https://sites.google.com/view/waqasliazhar/courses

Course Basics				
Credit Hours	3			
Lecture(s)	Nbr of Lec(s) Per Week	2	Duration	75 mins.
Recitation/Lab (per week)	Nbr of Lec(s) Per Week		Duration	
Tutorial (per week)	Nbr of Lec(s) Per Week		Duration	

Course Distribution	
Core	
Elective	
Open for Student Category	
Close for Student Category	

COURSE DESCRIPTION

This course covers the fundamentals of Differential Geometry: Local and global geometry of curves and surfaces, Frenet frames, Total curvature, Minimal surfaces, Geodesics, Gaussian curvature, Differentiable manifolds, Tangent bundles and Riemannian manifolds.

COURSE PREREQUISITE(S)

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Introduction to Analysis 1 (MATH 205).

COURSE OBJECTIVES

Upon successful completion, students will have the knowledge and skills to:

1. **Explain the concepts and language of differential geometry and its role in modern mathematics.**
2. **Analyse and solve complex problems using appropriate techniques from differential geometry with mathematical rigour.**

Learning Outcomes

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[CLO1]	**Local and global geometry of plane curves.**
[CLO2]	**Local geometry of surfaces.**
[CLO3]	**Global geometry of surfaces and manifolds.**
[CLO4]	**Study of Differential forms and Riemannian manifolds**

Grading Breakup and Policy
Assignment(s): 30% Midterm Examination: 30% Final Examination: 40%

Examination Detail	
Midterm Exam	Yes/No: Yes Combine Separate: Duration: Preferred Date: Exam Specifications:
Final Exam	Yes/No: Yes Combine Separate: Duration: Exam Specifications:

COURSE OVERVIEW

Lecture	Topics	Recommended Readings	Objectives/ Application
• 1-2	**Local theory of curves** Frenet curves, Plane and space curves, curvature and torsion, Frenet Formulae.	**WK 2.1**	**[CLO1]**
• 3-4	**Generalized Frenet Formulae** Fundamental theorem of the local theory of curves. The global theory of curves. Polar angle function, winding number and the rotation index.	**WK 2.2**	**[CLO1]**
• 5-6	**Total curvature and Total absolute curvature** Four vertex theorem, total absolute curvature of space curves.	**WK 2.3**	**[CLO1]**
• 7-8	**Local theory of surfaces and introduction to Manifolds** Surface elements and first fundamental form. Orientable surfaces.	**WK 3A**	**[CLO2]**
• 9-10	**Local theory of surfaces** Gauss map and curvature of surfaces. Second and third fundamental forms.	**WK 3B**	**[CLO2]**

• 11-12	**Local theory of surfaces** Principal and Gaussian curvature. Surfaces of rotation and ruled surfaces.	**WK 3B**	**[CLO2]**
• 13-14	**Minimal surface** Minimal surface problem. Conformal parametrizations.	**WK 3B**	**[CLO2]**
• 15-16	**The intrinsic geometry of surfaces** The covariant derivative and Christoffel symbols.	**WK 4B**	**[CLO3]**
• 17-18	**Gaussian Equation** Gaussian equation and geodesics.	**WK 4C**	**[CLO3]**
• 19-20	**Gaussian curvature in special parameters** Orthogonal parameters and geodesic parallel coordinates.	**WK4E**	**[CLO3]**
• 21-22	**Gauss Bonnet Theorem** Differential forms and Gauss Bonnet Theorem.	**WK 4F**	**[CLO3]**
• 23-24	**Differentiable manifolds** Structures on a manifold, charts, atlas.	**WK 5A**	**[CLO4]**
• 25-26	**The tangent space** Tangent space, chain rule of differential on a manifold.	**WK 5B**	**[CLO4]**
• 27-28	**Riemannian manifolds** Riemannian metrics and Riemannian manifolds	**WK5C**	**[CLO4]**

Textbook(s)/Supplementary Readings

- [WK] Kuhnel, Wolfgang. *Differential Geometry: Curves – Surfaces – Manifolds*. Student mathematical library, vol. 16. Providence, RI: American Mathematical Society, 2002. ISBN: 9780821826560.
- Spivak, Michael. *A Comprehensive Introduction to Differential Geometry*. Vol. 2. Boston, MA: Publish or Perish, 1999. ISBN: 9780914098713.
- [WB] William M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, second revised edition, Academic Press, 2002.