Mid-term exam for the course "Differential geometry", October 13, 2023

1. Let $S \subset \mathbb{R}^3$ be a connected subset of $\mathbb{R}^3$. Assume that there exists a family $\{S_\alpha\}$ of surfaces in $\mathbb{R}^3$ such that

(i) every $S_\alpha \subseteq S$ is open in S and
(ii) $S = \bigcup_\alpha S_\alpha$.

Prove that S is a surface.

2. Determine the tangent plane at each point of the hyperbolic paraboloid $S \subset \mathbb{R}^3$ of equation $x^2 - y^2 - z = 0$ using the global parametrization $\varphi \colon \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\varphi(x_1, x_2) = (x_1, x_2, x_1^2 - x_2^2) \, .$$