IMM-LUMS LAHORE
ADVANCED TOPOLOGY

Assignment 1

Exercise 1. Let X be a set and fix $x_0 \in X$. Prove that the following collections are topologies on X:

$$\tau_1 := \{A \subseteq X : x_0 \in A\} \cup \{\varnothing\},$$
$$\tau_2 := \{A \subseteq X : x_0 \notin A\} \cup \{X\}.$$

Are (X, τ_1) and (X, τ_2) homeomorphic?

Exercise 2. Consider X, Y topological spaces and $X \times Y$ with the product topology. Let $A \subseteq X$, $B \subseteq Y$. Prove that $\overline{A \times B} = \overline{A} \times \overline{B}$.

Exercise 3. Prove that a topological space X is Hausdorff if and only if the diagonal $D := \{(x, x) : x \in X\} \subseteq X \times X$ is closed, where $X \times X$ is endowed with the product topology.

Exercise 4. A subset D of a topological space X is called *dense* if $\overline{D} = X$. Prove that a subset D is dense if and only if $D \cap A \neq \varnothing$ for any open set A. Prove that, if $f : X \to Y$ is a continuous surjective function and $D \subseteq X$ is dense in X, then $f(D)$ is dense in Y.

Date: Due by: January 22nd, 2024.