

**IMM-LUMS LAHORE  
ADVANCED TOPOLOGY**

ASSIGNMENT 2

**Exercise 1.** Let  $(X, d)$  be a metric space with the induced topology. Consider  $X \times X$  with the product topology and  $\mathbb{R}$  with the Euclidean topology. Prove that  $d: X \times X \rightarrow \mathbb{R}$  is continuous.

**Exercise 2.** Consider the function  $\text{sgn}: \mathbb{R} \rightarrow \{-1, 0, 1\}$  defined by  $\text{sgn}(0) = 0$  and  $\text{sgn}(x) = x/|x|$  for  $x \neq 0$ . Describe the quotient topology on  $\{-1, 0, 1\}$ .

**Exercise 3.** On  $\mathbb{R}$ , consider the equivalence relation  $x \sim y$  if and only if  $x - y \in \mathbb{Q}$ . Describe the quotient topology on  $\mathbb{R}/\sim$ .

**Exercise 4.** Consider the real line  $\mathbb{R}$  endowed with the co-countable topology (namely, the proper closed subsets are the subsets with cardinality at most countable). Which subsets are connected? Is it path-connected?

**Exercise 5.** Consider  $\mathbb{R}$  endowed with the Euclidean topology. Prove that there exists no continuous function  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(\mathbb{Q}) \subseteq \mathbb{R} \setminus \mathbb{Q}$  and  $f(\mathbb{R} \setminus \mathbb{Q}) \subseteq \mathbb{Q}$ .